RISK ANALYSIS AND THE EXPECTED VALUE OF PERFECT INFORMATION

Michael R. Middleton, University of San Francisco

ABSTRACT

This paper describes a procedure for determining the expected value of perfect information in computer-based risk analysis models. Decision tree diagrams illustrate the procedure and other methods, including discrete approximations and Monte Carlo simulation.

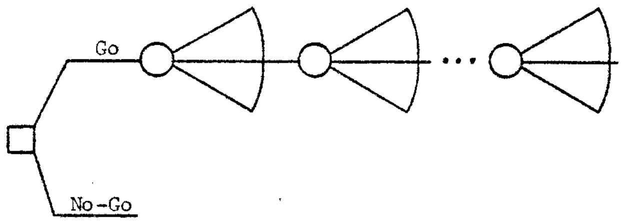
INTRODUCTION

Risk analysis, originally described by Hertz((1)), has become a popular method for evaluating uncertainty in capital investment projects. The Monte Carlo simulation feature is included in many of the commercial computer-based financial planning systems, and most business school students are introduced to the risk analysis methodology in their finance or management science courses. In this paper we propose a procedure for determining the expected value of perfect information (EVPI) in computer-based risk analysis models. We restrict our discussion to the type of problem where risk analysis is usually employed: the go vs. no-go decision for an individual project.

The risk analysis method and our proposed procedure encompass the three phases of the decision analysis cycle discussed by Howard((3)). First, the deterministic phase specifies a deterministic model of the decision problem. In risk analysis applications this model is often a set of accounting relationships for determining cash flows and net present value (NPV). For example, in a decision problem concerning the introduction of a new product, the annual cash flows may depend upon quantity sold, selling price, raw material cost, and other factors. Then deterministic sensitivity analysis can identify critical variables that warrant explicit consideration of uncertainty.

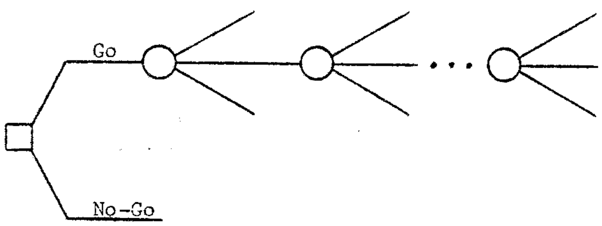
Second, the probabilistic phase involves assessment of probability distributions for the critical variables. Figure 1 shows the probabilistic model in decision tree form, where event fans are used to represent continuous or many-valued random variables. In each trial of the risk analysis method a random value is generated from each probability distribution, thereby selecting a random path through the tree. Then the deterministic accounting model computes the NPV associated with the endpoint of that path. After a large number of trials the representative sample of endpoint values can be summarized as a risk profile, i.e., a probability distribution of the uncertain NPV associated with the project. (In many cases the decision maker may be able to make the go vs. no-go choice directly by examining the risk profile and its summary measures, such as the mean, standard deviation, etc. Other cases may require assessing the decision maker's risk preference curve and computing expected utility.)

FIGURE 1



Alternatively, the risk profile can be calculated using discrete approximations instead of Monte Carlo simulation. Figure 2 shows a decision tree where each continuous random variable from Figure 1 has been approximated by three discrete values. For example, if there are five random variables, then the "go" alternative in Figure 2 has 243 endpoints. As before, the deterministic accounting model computes NPV for each endpoint. If each discrete approximation does not employ equally likely values, then the joint probability for each path must also be calculated individually in order to develop the risk profile.

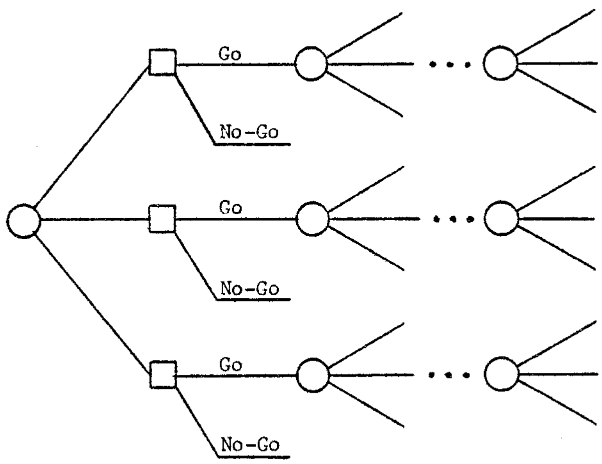
FIGURE 2



Third, the informational phase of the decision analysis cycle involves finding the potential value of information regarding each random variable. In the following discussion we will assume that the decision maker is risk neutral, so EVPI is the appropriate measure for screening real opportunities for information gathering. Holloway ((2, p. 349)) defines EVPI as the difference between two expected values, i.e., the expected value if perfect information could be obtained minus the expected value of the best alternative without information; in risk analysis problems the latter expected value will be either the mean of the "go" alternative's NPV risk profile or the value assigned to the "no-go" alternative, whichever is greater. The hypothetical situation where perfect information can be obtained is shown in Figure 3.

One random variable is selected, and the event fork for that variable is moved to the front of the decision tree. Thus, Figure 3 illustrates the situation where the value of the variable is known before the go vs. no-go choice must be made. Since the decision maker is uncertain about which specific value of the variable will be revealed by the perfect information, the hypothetical situation is summarized by an expected value computation. Then EVPI equals the expected value of Figure 3 minus the expected value of Figure 2. Computer programs are available for performing both the expected value computations and the automatic rearrangement of event nodes in discrete decision trees((4)).

FIGURE 3

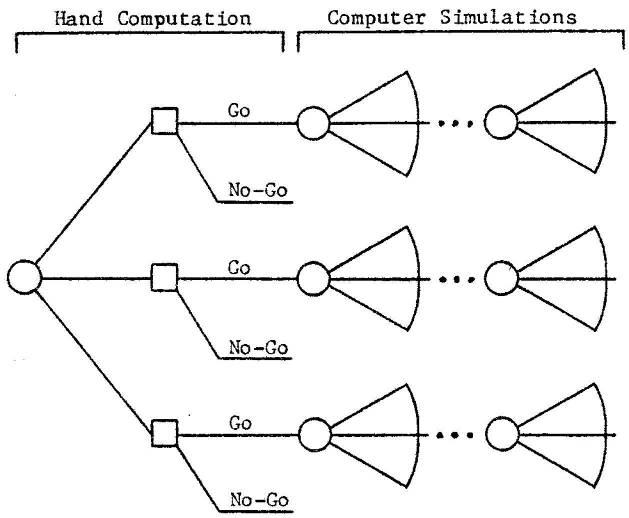


SIMULATION AND EVPI

Many of the EVPI computations can be performed using the Monte Carlo simulation feature of most computer-based financial planning systems, but some of the calculations must be performed by the user. One approach is illustrated in Figure 4. The user selects one of the continuous random variables from the original risk analysis problem shown in Figure 1. The continuous distribution is approximated by several discrete values. For each discrete value of that variable the user makes a separate simulation run. The variable of interest is constant for each run, but values for all other variables are randomly selected from their respective continuous distributions. After the simulation runs are completed, the user compares the mean of the "go" alternative with the value of the "no-go" alternative, and the expected value of the left-hand event node in Figure 4 is computed. Then EVPI equals the expected value of Figure 4 minus the expected value of Figure 1.

One could write a computer program which would perform all of the calculations indicated in Figure 4 for each random variable, but we propose a different procedure that should be more efficient in most computing environments. This procedure does not require additional simulation runs. Instead, it uses the results of the initial simulation run (Figure 1) for computing the EVPI for each variable. The simulation results are stored temporarily in a two-dimensional array. One row is needed for each trial from the initial simulation run. One column is used for each variable in the model; one extra column contains the NPV associated with each random trial. The initial entries in the array are the randomly selected values for each variable on each trial.

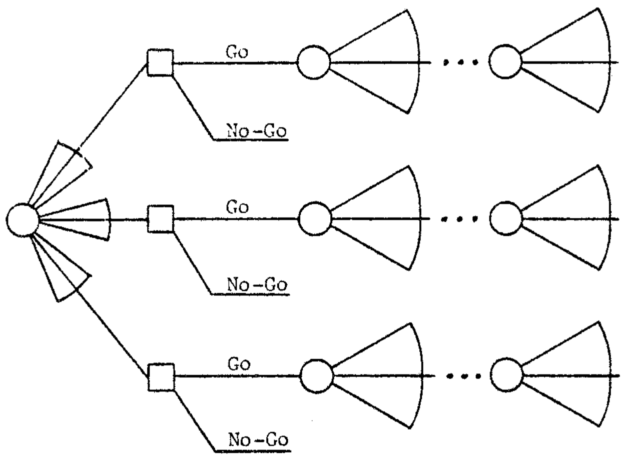
FIGURE 4



A subprogram uses the array values to compute EVPI for each variable of interest. The EVPI computations for a specific variable involve only the values of that variable and the associated NPV for each trial; i.e., only two columns of the array are needed for each EVPI computation. Figure 5 illustrates the case where the continuous distribution of a selected variable is divided into three brackets for the EVPI computations. For example, values up to the 33 percentile might be labelled "low", values between the 33 percentile and the 67 percentile "medium", and values above the 67 percentile "high". Approximately one-third of the values from a large simulation run should fall within each bracket.

For each bracket the mean of the associated NPV entries is computed. The mean is compared with the value of the "no-go" alternative, and the maximum is selected. The resulting values (at the decision node squares in Figure 5) are summarized in an expected value calculation, using the probability associated with each bracket. Finally, the EVPI equals the expected value of Figure 5 minus the expected value of Figure 1.

FIGURE 5



CONCLUSION

There are two possible advantages of this procedure compared with the method of Figure 4. First, the total number of simulation trials can be allocated to the initial run, thereby obtaining a more accurate sample for the risk profile. Second, the procedure incorporates the concept of blocking, i.e., the same paths through the tree are used to compute both expected values in the EVPI calculation. Future research may investigate how the estimates of EVPI depend upon sample size and the number brackets. Computer programs in Applesoft BASIC applying the three methods to a three-variable example are available from the author.

REFERENCES

((1)) Hertz, D. B., "Risk Analysis in Capital Investment", Harvard Business Review, January-February 1964, pp. 95-106.

((2)) Holloway, C. A., Decision Making Under Uncertainty: Models and Choices. Prentice-Hall Englewood Cliffs, New Jersey, 1979.

((3)) Howard, R. A., "The Foundations of Decision Analysis", IEEE Transactions on Systems Sciences and Cybernatics, Vol. SSC-4 (1968), pp. 211-219.

((4)) Rousseau, W. F. and North, D. W., A User's Manual for TREE: A Program for the Analysis of Decision and Probability Trees. Stanford Research Institute, Menlo Park, California, 1971.

PROCEEDINGS

AMERICAN INSTITUTE FOR DECISION SCIENCES
ELEVENTH ANNUAL MEETING
WESTERN REGIONAL CONFERENCE

San Diego Hilton Hotel
San Diego, California
March 17-19, 1982

EDITORS
Daniel G. Brooks
William A. Verdini
Arizona State University
Tempe, Arizona