## Strategies in Decision Trees

### 16.1 STRATEGY

A strategy specifies an initial choice and any subsequent choices to be made by the decision maker. The subsequent choices usually depend upon events. The specification of a strategy must be comprehensive; if the decision maker gives the strategy to a colleague, the colleague must know exactly which choice to make at each decision node.

Most decision problems have many possible strategies, and a goal of the analysis is to determine the optimal strategy, taking into account the decision maker's risk attitude. There are four strategies in the DriveTek problem. One of the strategies is: Prepare the proposal; if not awarded the contract, stop; if awarded the contract, try the magnetic method; if the magnetic method is successful, stop; if the magnetic method fails, use the mechanical method. The four strategies will be discussed in detail below.

### 16.2 PAYOFF DISTRIBUTION

Each strategy has an associated payoff distribution, sometimes called a risk profile. The payoff distribution of a particular strategy is a probability distribution showing the probability of obtaining each terminal value associated with a particular strategy.

In decision tree models, the payoff distribution can be shown as a list of possible payoff values, $x$, and the discrete probability of obtaining each value, $\mathrm{P}(\mathrm{X}=\mathrm{x})$, where X represents the uncertain terminal value associated with a strategy. Since a strategy specifies a choice at each decision node, the uncertainty about terminal values depends only on the occurrence of events. The probability of obtaining a specific terminal value equals the product of the probabilities on the event branches on the path leading to the terminal node.

### 16.3 DRIVETEK STRATEGIES

In this section each strategy of the DriveTek problem is described by a shorthand statement and a more detailed statement. The possible branches following a specific strategy are shown in decision tree form, and the payoff distribution is shown in a table with an explanation of the probability calculations.

Strategy 1 (Mechanical): Prepare; if awarded, use mechanical.
Details: Prepare the proposal; if not awarded the contract, stop (payoff $=-\$ 50,000$ ); if awarded the contract, use the mechanical method (payoff $=\$ 80,000)$.

Figure 16.1 Bold Format Branches for Mechanical Strategy


Figure 16.2 Payoff Distribution for Mechanical Strategy

| Value, $x$ | Probability <br> $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |
| :--- | :--- |
| $\$ 80,000$ |  |
| $-\$ 50,000$ |  |

Strategy 2 (Electronic): Prepare; if awarded, try electronic.
Details: Prepare the proposal; if not awarded the contract, stop (payoff $=-\$ 50,000$ ); if awarded the contract, try the electronic method; if the electronic method is successful, stop (payoff = $\$ 150,000)$; if the electronic method fails, use the mechanical method (payoff $=\$ 30,000$ ).

Figure 16.3 Bold Format Branches for Electronic Strategy


Figure 16.4 Payoff Distribution for Electronic Strategy

| Value, $x$ | Probability <br> $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |  |
| :--- | :--- | :--- |
| $\$ 150,000$ | 0.25 |  |
| $\$ 30,000$ | 0.25 | $=0.5 * 0.5$ |
| $-\$ 50,000$ | $\frac{0.50}{1.00}$ | $=0.5 * 0.5$ |

Strategy 3 (Magnetic): Prepare; if awarded, try magnetic.
Details: Prepare the proposal; if not awarded the contract, stop (payoff $=-\$ 50,000$ ); if awarded the contract, try the magnetic method; if the magnetic method is successful, stop (payoff $=\$ 120,000$ ); if the magnetic method fails, use the mechanical method (payoff = \$0).

Figure 16.5 Bold Format Branches for Magnetic Strategy


Figure 16.6 Payoff Distribution for Magnetic Strategy

| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |  |
| :---: | :---: | :---: |
| \$120,000 | 0.35 | $=0.5 * 0.7$ |
| \$0 | 0.15 | $=0.5 * 0.3$ |
| -\$50,000 | 0.50 |  |
|  | 1.00 |  |

## Strategy 4 (Don't): Don't.

Details: Don't prepare the proposal (payoff $=\$ 0$ ).

Figure 16.7 Bold Format Branches for Don't Strategy


Figure 16.8 Payoff Distribution for Don't Strategy
Value, $x$

$\$ 0$$\frac{$|  Probability  |
| :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |}{1.00}

### 16.4 STRATEGY CHOICE

Since each strategy can be characterized completely by its payoff distribution, selecting the best strategy becomes a problem of choosing the best payoff distribution.

One approach is to make a choice by direct comparison of the payoff distributions.

Figure 16.9 Payoff Distributions for Four Strategies

| Strategy 1 (Mechanical) |  | Strategy 2 (Electronic) |  |
| :---: | :---: | :---: | :---: |
| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |
| $\begin{aligned} & \$ 80,000 \\ & -\$ 50,000 \end{aligned}$ | 0.50 | \$150,000 | 0.25 |
|  | 0.50 | \$30,000 | 0.25 |
|  | 1.00 | -\$50,000 | 0.50 |
|  |  |  | 1.00 |
| Strategy 3 (Magnetic) |  | Strategy 4 (Don't) |  |
| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |
| \$120,000 | 0.35 | \$0 | 1.00 |
| $\begin{aligned} & \$ 0 \\ & -\$ 50,000 \end{aligned}$ | 0.15 |  | 1.00 |
|  | 0.50 |  |  |
|  | 1.00 |  |  |

Another approach for making choices involves certain equivalents.

### 16.5 CERTAIN EQUIVALENT

A certain equivalent is a certain payoff value which is equivalent, for the decision maker, to a particular payoff distribution. If the decision maker can determine his or her certain equivalent for the payoff distribution of each strategy, then the optimal strategy is the one with the highest certain equivalent.

The certain equivalent is the minimum selling price for a payoff distribution; it depends on the decision maker's personal attitude toward risk. A decision maker may be risk preferring, risk neutral, or risk avoiding.

If the terminal values are not regarded as extreme (relative to the decision maker's total assets), if the decision maker will encounter other decision problems with similar payoffs, and if the decision maker has the attitude that he or she will "win some and lose some," then the decision maker's attitude toward risk may be described as risk neutral.

If the decision maker is risk neutral, the expected value is the appropriate certain equivalent for choosing among the strategies. Thus, for a risk neutral decision maker, the optimal strategy is the one with the highest expected value.

The expected value of a payoff distribution is calculated by multiplying each terminal value by its probability and summing the products. The expected value calculations for each of the four strategies of the DriveTek problem are shown below.

Figure 16.10 Payoff Distributions and Expected Values

## Strategy 1 (Mechanical)

| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | x * P(X=x) |
| :---: | :---: | :---: |
| \$80,000 | 0.50 | \$40,000 |
| -\$50,000 | 0.50 | -\$25,000 |
|  |  | \$15,000 |

Strategy 2 (Electronic)

| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | x * $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |
| :---: | :---: | :---: |
| \$150,000 | 0.25 | \$37,500 |
| \$30,000 | 0.25 | 7,500 |
| -\$50,000 | 0.50 | -\$25,000 |
|  |  | \$20,000 |

Strategy 3 (Magnetic)

| Value, x | Probability $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | x * $\mathrm{P}(\mathrm{X}=\mathrm{x})$ |
| :---: | :---: | :---: |
| \$120,000 | 0.35 | \$42,000 |
| \$0 | 0.15 | \$0 |
| -\$50,000 | 0.50 | -\$25,000 |
|  |  | \$17,000 |



The four strategies of the DriveTek problem have expected values of $\$ 15,000, \$ 20,000, \$ 17,000$, and $\$ 0$. Strategy 2 (Electronic) is the optimal strategy with expected value $\$ 20,000$.

A risk neutral decision maker's choice is based on the expected value. However, note that if strategy 2 (Electronic) is chosen, the decision maker does not receive $\$ 20,000$. The actual payoff will be $\$ 150,000, \$ 30,000$, or $-\$ 50,000$, with probabilities shown in the payoff distribution.

The rollback method used by TreePlan identified strategy 2 (Electronic) as optimal. The rollback value on the initial branch of the optimal strategy is $\$ 20,000$, which must be the same as the expected value for the payoff distribution of strategy 2 . Some of the intermediate calculations for the rollback method differ from the calculations for the payoff distributions, but both approaches identify the same optimal strategy with the same initial expected value. For decision trees with a large number of strategies, the rollback method is more efficient.

### 16.6 BRANDON DECISION TREE PROBLEM

Brandon Appliance Corporation, a predominant producer of microwave ovens, is considering the introduction of a new product. The new product is a microwave oven that will defrost, cook, brown, and boil food as well as sense when the food is done.

Brandon must decide on a course of action for implementing this new product line. An initial decision must be made to (1) nationally distribute the product from the start, (2) conduct a marketing test first, or (3) not market the product at all. If a marketing test is conducted, Brandon will consider the result and then decide whether to abandon the product line or make it available for national distribution.

The finance department has provided some cost information and probability assignments relating to this decision. The preliminary costs for research and development have already been incurred and are considered irrelevant to the marketing decision. A success nationally will increase profits by $\$ 5,000,000$, and failure will reduce them by $\$ 1,000,000$, while abandoning the product will not affect profits. The test market analysis will cost Brandon an additional $\$ 35,000$.

If a market test is not performed, the probability of success in a national campaign is 60 percent. If the market test is performed, the probability of a favorable test result is 58 percent. With favorable test results, the probability for national success is approximately 93 percent. However, if the test results are unfavorable, the national success probability is approximately 14 percent.

## Decision Tree Strategies

Brandon Appliance Corporation must decide on a course of action for implementing this new microwave oven. An initial decision must be made to (1) nationally distribute the product from the start, (2) conduct a marketing test first, or (3) not market the product at all. If a marketing test is conducted, Brandon will consider the result and then decide whether to abandon the product line or make it available for national distribution. The following decision tree is based on information about cash flows and probability assignments.

Figure 16.11 Brandon Decision Tree


In a decision tree model, a strategy is a specification of an initial choice and any subsequent choices that must be made by the decision maker.

How many strategies are there in the Brandon problem?
Describe each strategy.

Figure 16.12 Strategy 1: National


Figure 16.13 Strategy 2: Test; if Favorable, National; if Unfavorable, National


Figure 16.14 Strategy 3: Test; if Favorable, National; if Unfavorable, Don't


Figure 16.15 Strategy 4: Test; if Favorable, Don't; if Unfavorable, National


Figure 16.16 Strategy 5: Test; if Favorable, Don't; if Unfavorable, Don't


Figure 16.17 Strategy 6: Don't


