Value of Information in Decision Trees

19

19.1 VALUE OF INFORMATION

Useful concept for

Evaluating potential information-gathering activities

Comparing importance of multiple uncertainties

19.2 EXPECTED VALUE OF PERFECT INFORMATION

Several computational methods

Flipping tree, moving an event set of branches, appropriate for any decision tree

Payoff table, most appropriate only for single-stage tree (one set of uncertain outcomes with no subsequent decisions)

Expected improvement

All three methods start by determining Expected Value Under Uncertainty, EVUU, which is the expected value of the optimal strategy without any additional information.

To use these methods, you need (a) a model of your decision problem under uncertainty with payoffs and probabilities and (b) a willingness to summarize a payoff distribution (payoffs with associated probabilities) using expected value.

The methods can be modified to use certain equivalents for a decision maker who is not risk neutral.

Expected Value of Perfect Information, Reordered Tree

Figure 19.1 Structure, Cash Flows, Endpoint Values, and Probabilities

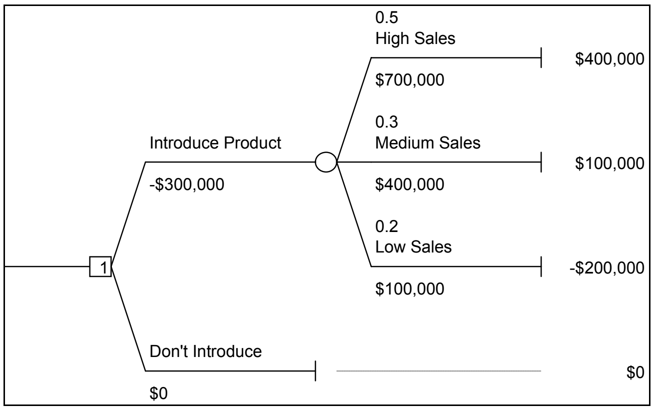
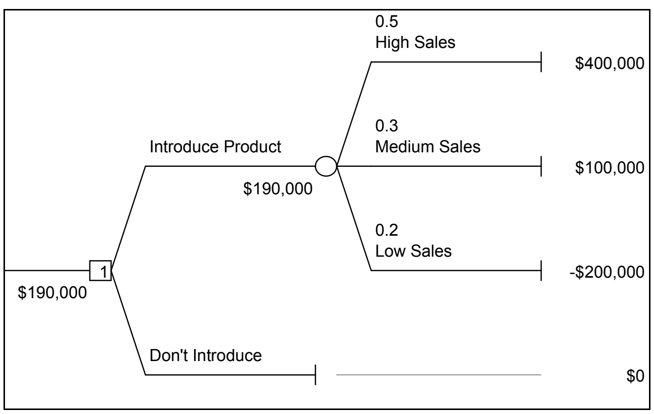
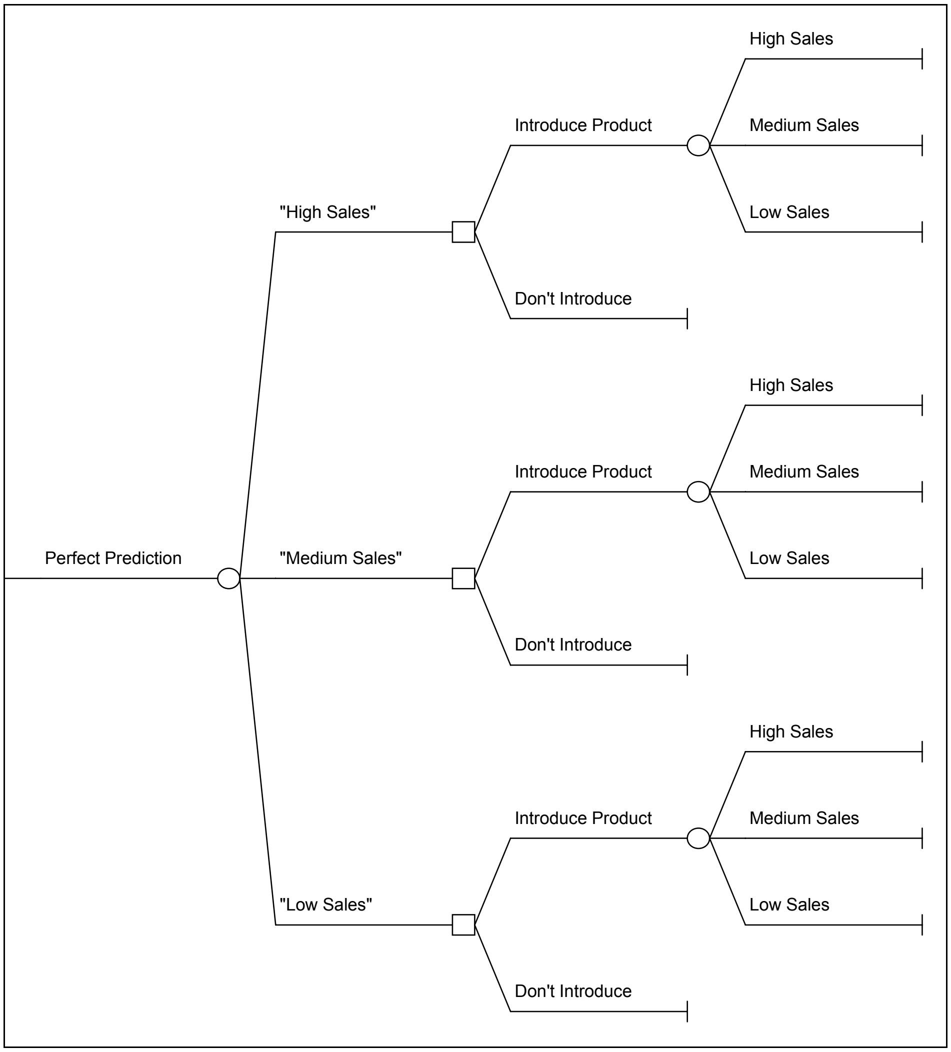


Figure 19.2 Rollback Expected Values



The two figures above show what is called the *prior* problem, i.e., the decision problem under uncertainty before obtaining any additional information.

Figure 19.3 Structure Using Perfect Prediction

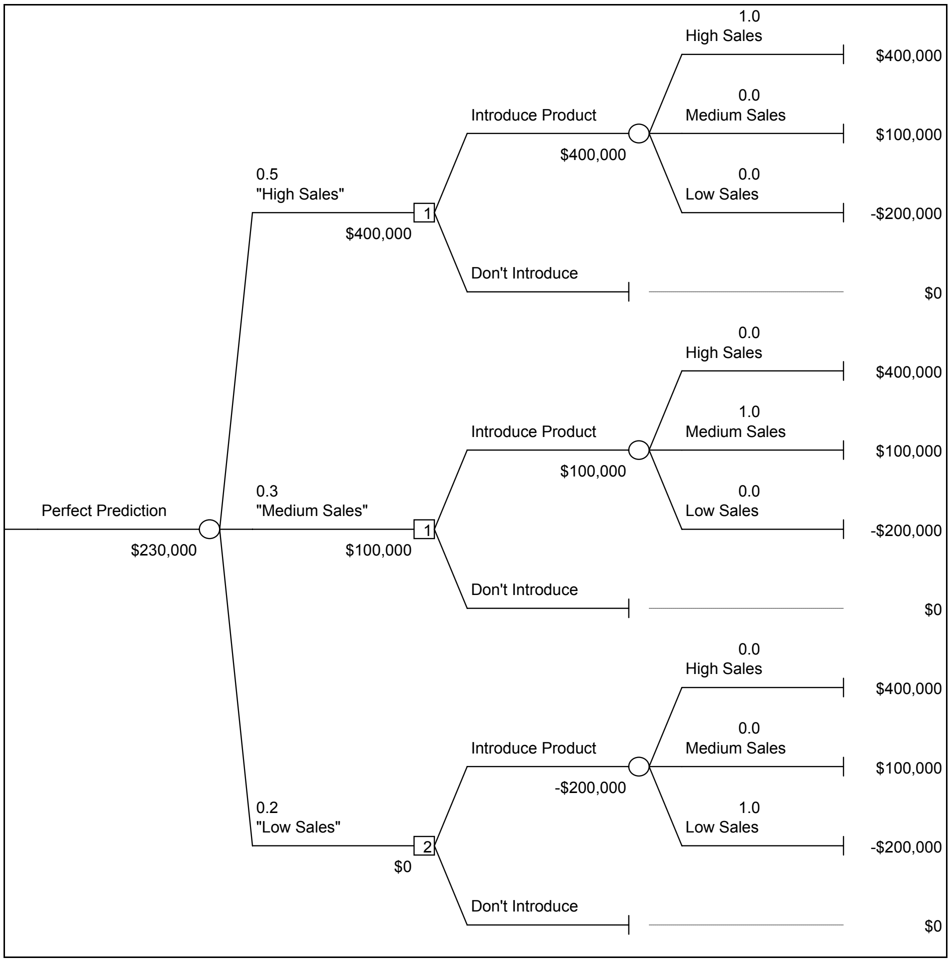


Before you get a perfect prediction, you are uncertain about what that prediction will be.

If you originally think the probability of High Sales is 0.5, then you should also think the probability is 0.5 that a perfect prediction will tell you that sales will be high.

After you get a prediction of "High Sales," the probability of actually having high sales is 1.0.

Figure 19.4 Rollback Using Free Perfect Prediction



EVUU: Expected Value Under Uncertainty

the expected value of the best strategy without any additional information

EVPP Expected Value using a (free) Perfect Prediction

EVPI Expected Value of Perfect Information

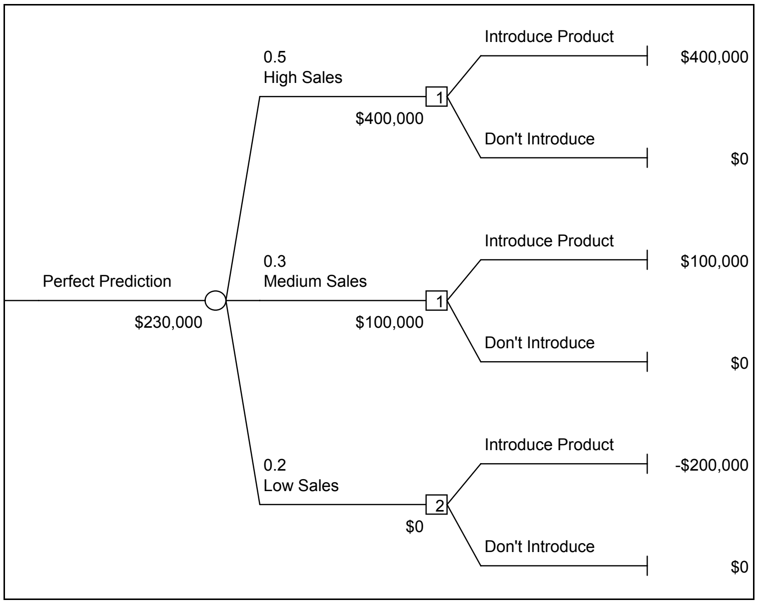
EVPI = EVPP – EVUU

In this example, EVPI = $230,000 – $190,000 = $40,000

For a perfect prediction, the information message "Low Sales" is the same as the event Low Sales, so the detailed structure shown above is not needed.

A shortcut approach is to "flip" the original decision tree, shown in Figure 19.2, rearranging the order of the decision node and event node, to obtain the tree shown below.

Figure 19.5 Shortcut EVPP



Expected Value of Perfect Information, Payoff Table

This method is most appropriate only for a single-stage decision tree (one set of uncertain outcomes with no subsequent decisions).

Figure 19.6 Payoff Table for Prior Problem with Expected Values

		Alternatives	
Probability	Event	Introduce	Don't
0.5	High Sales	$400,000	$0
0.3	Medium Sales	$100,000	$0
0.2	Low Sales	-$200,000	$0
	Expected Value	$190,000	$0

For each row in the body of the payoff table, if you receive a perfect prediction that the event in that row will occur, which alternative would you choose and what would your payoff be?

Before you receive the prediction, you don't know which of the payoffs you will receive (either $400,000 or $100,000 or $0), so you summarize the payoff distribution using expected value, EVPP.

Figure 19.7 Payoff Table with EVPP

		Alternatives			Payoff Using
Probability	Event	Introduce	Don't		Perfect Prediction
0.5	High Sales	$400,000	$0		$400,000
0.3	Medium Sales	$100,000	$0		$100,000
0.2	Low Sales	-$200,000	$0		$0
	Expected Value	$190,000	$0		$230,000
		EVUU			EVPP

EVPI = $230,000 – $190,000 = $40,000

Expected Value of Perfect Information, Expected Improvement

Like the payoff table method, this method is most appropriate only for a single-stage decision tree.

(1) Use the prior decision tree or prior payoff table to find EVUU (the expected value of the best strategy without any additional information).

(2) If you are committed to the best strategy, consider each outcome of the uncertain event and whether you would change your choice if you received a perfect prediction that the event was going to occur.

In the example, you would not change your choice if you are told that sales will be high or medium. However, if you are told that sales will be low, you would change your choice from Introduce to Don't.

(3) Determine how much your payoff will improve in each of the cases.

In the example, your payoff will not improve if you are told that sales will be high or medium, but your payoff will improve by $200,000 (from –$200,000 to $0) if you are told that sales will be low.

(4) Compute expected improvement associated with having the perfect prediction by weighting each improvement by its associated probability.

In the example, the improvements associated with a perfect prediction of high, medium, and low are $0, $0, and $200,000, respectively, with probabilities 0.5, 0.3, 0.2.

EVPI = Expected Improvement = 0.5*0 + 0.3*0 + 0.2*200,000 = $40,000

Expected Value of Perfect Information, Single-Season Product

Figure 19.8 Prior Problem, Four Alternatives and Three Outcomes

	A	B	C	D	E	F
1	**Single-Season Product**					
2						
3	**Data**					
4						
5	Price	$3.00				
6			Equip. Size			
7			None	Small	Medium	Large
8		Fixed Cost	$0	$1,000	$2,000	$3,000
9		Var. Cost	$0.00	$0.90	$0.70	$0.50
10		Capacity	0	4500	5500	6500
11						
12	**Payoff Table**					
13						
14			Equip. Size			
15	Prob.	Demand	None	Small	Medium	Large
16	0.3	3000	$0	$5,300	$4,900	$4,500
17	0.4	4000	$0	$7,400	$7,200	$7,000
18	0.3	5000	$0	$8,450	$9,500	$9,500
19						
20		Exp.Val.	$0	$7,085	$7,200	$7,000
21						
22						
23	C16 formula:		=(B5-C$9)*MIN(C$10,$B16)-C$8			
24	copied to C16:F18					
25						
26	C20 formula:		=SUMPRODUCT($A16:$A18,C16:C18)			
27	copied to C20:F20					

Figure 19.9 EVPP

	A	B	C	D	E	F	G	H	I
14			Equip. Size				Payoff Using		
15	Prob.	Demand	None	Small	Medium	Large	Perfect Prediction		
16	0.3	3000	$0	$5,300	$4,900	$4,500		$5,300	
17	0.4	4000	$0	$7,400	$7,200	$7,000		$7,400	
18	0.3	5000	$0	$8,450	$9,500	$9,500		$9,500	
19									
20		Exp.Val.	$0	$7,085	$7,200	$7,000		$7,400	
21									
22	H16 formula =MAX(C16:F16) copied to H16:H18								
23	C20 formula copied to H20								

EVPI = EVPP – EVUU = $7,400 – $7,200 = $200

Figure 19.10 Basic Probability Decision Tree

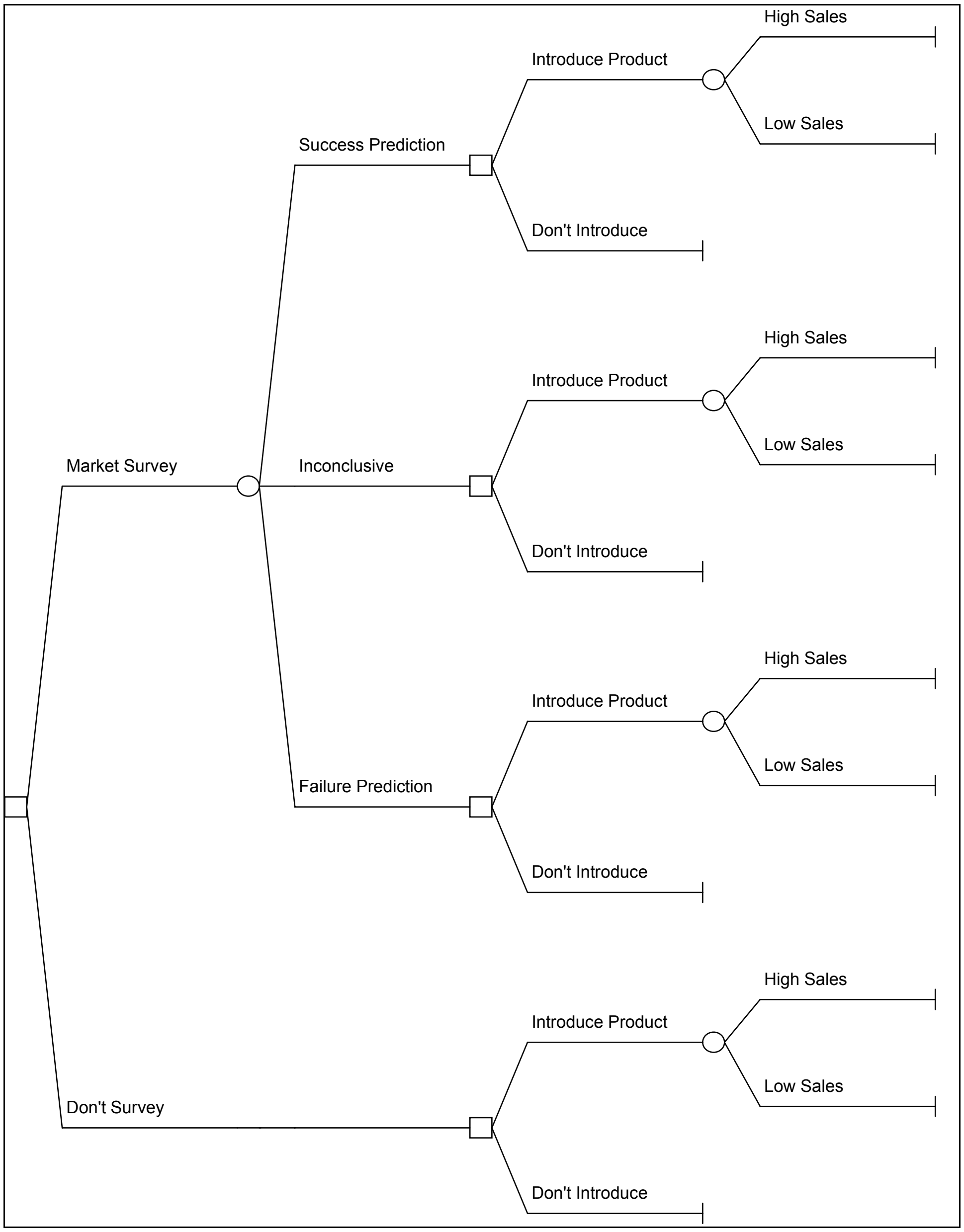
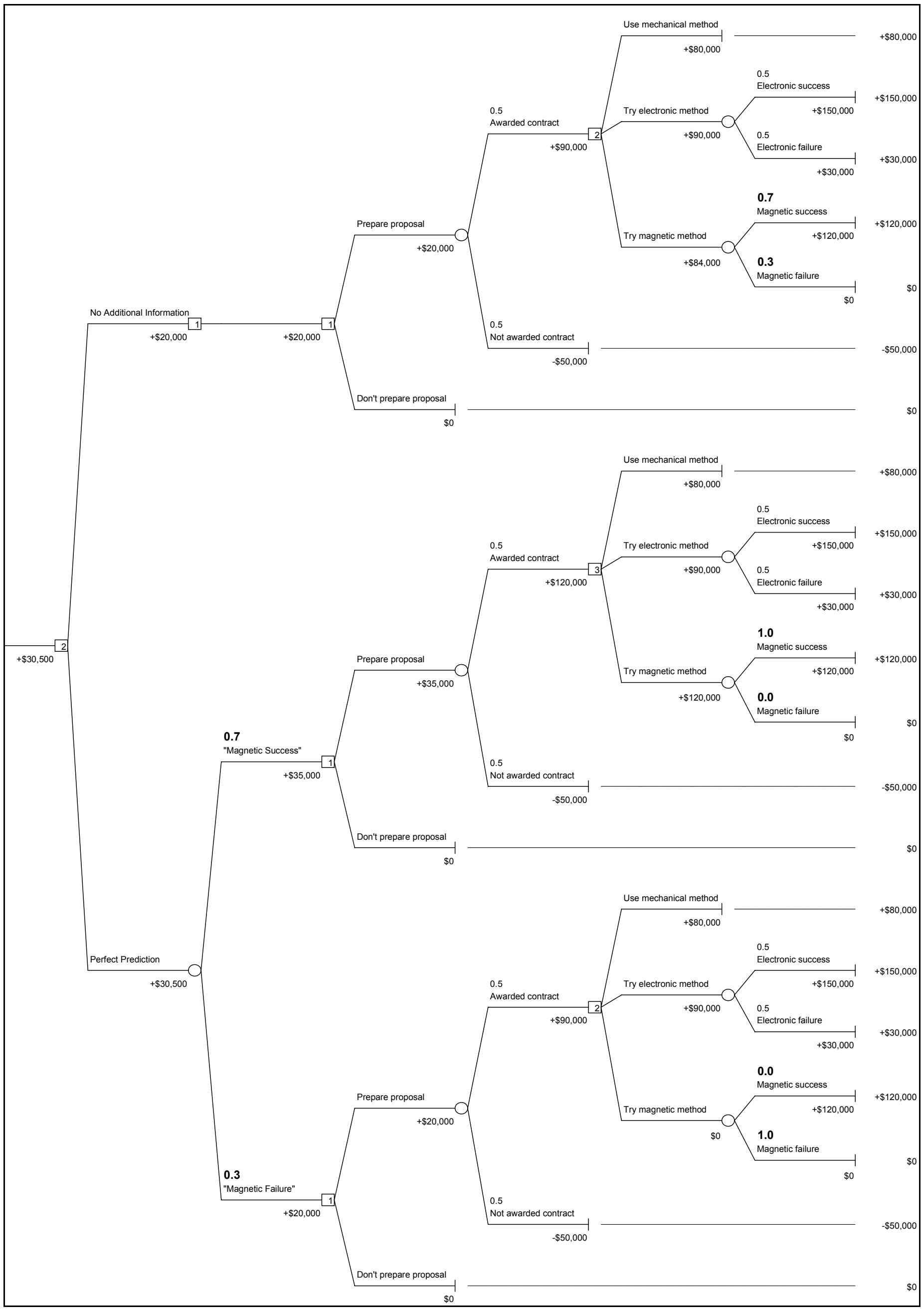


Figure 19.11 DriveTek EVPI Magnetic Success/Failure



19.3 DRIVETEK POST-CONTRACT-AWARD PROBLEM

DriveTek decided to prepare the proposal, and it turned out that they were awarded the contract. The $50,000 cost and $250,000 up-front payment are in the past. The current decision is to determine which method to use to satisfy the contract at minimum expected cost.

The following decision trees show costs for cash flows, terminal values, and rollback values. The rollback method uses TreePlan's option to minimize cost of immediate successors.

Figure 19.12 Costs for Cash Flows and Terminal Values

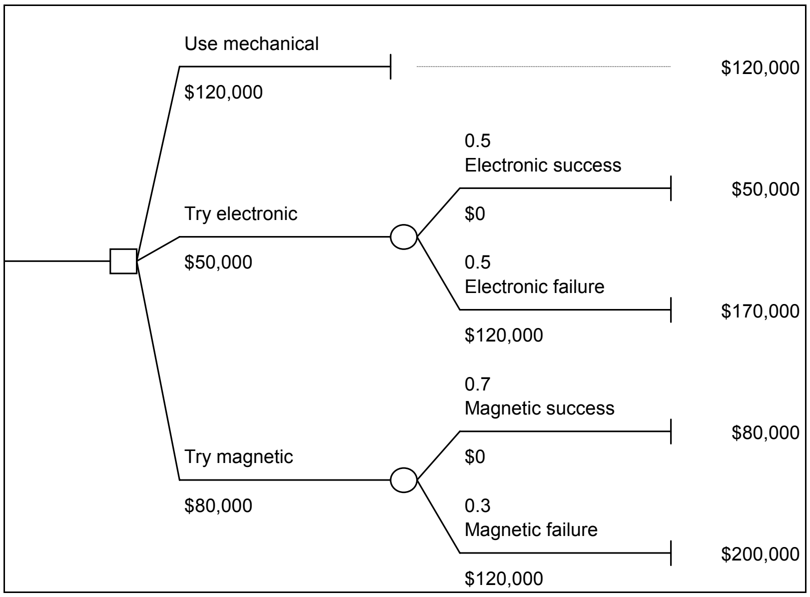
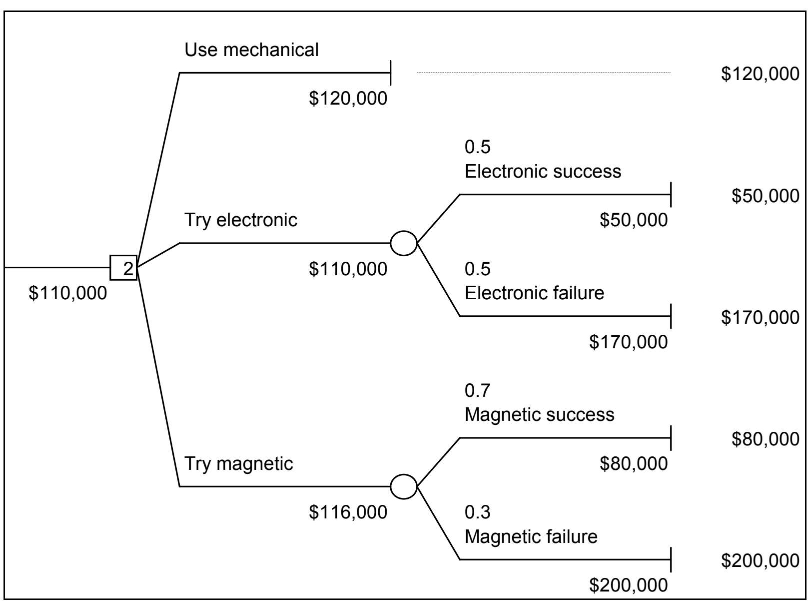
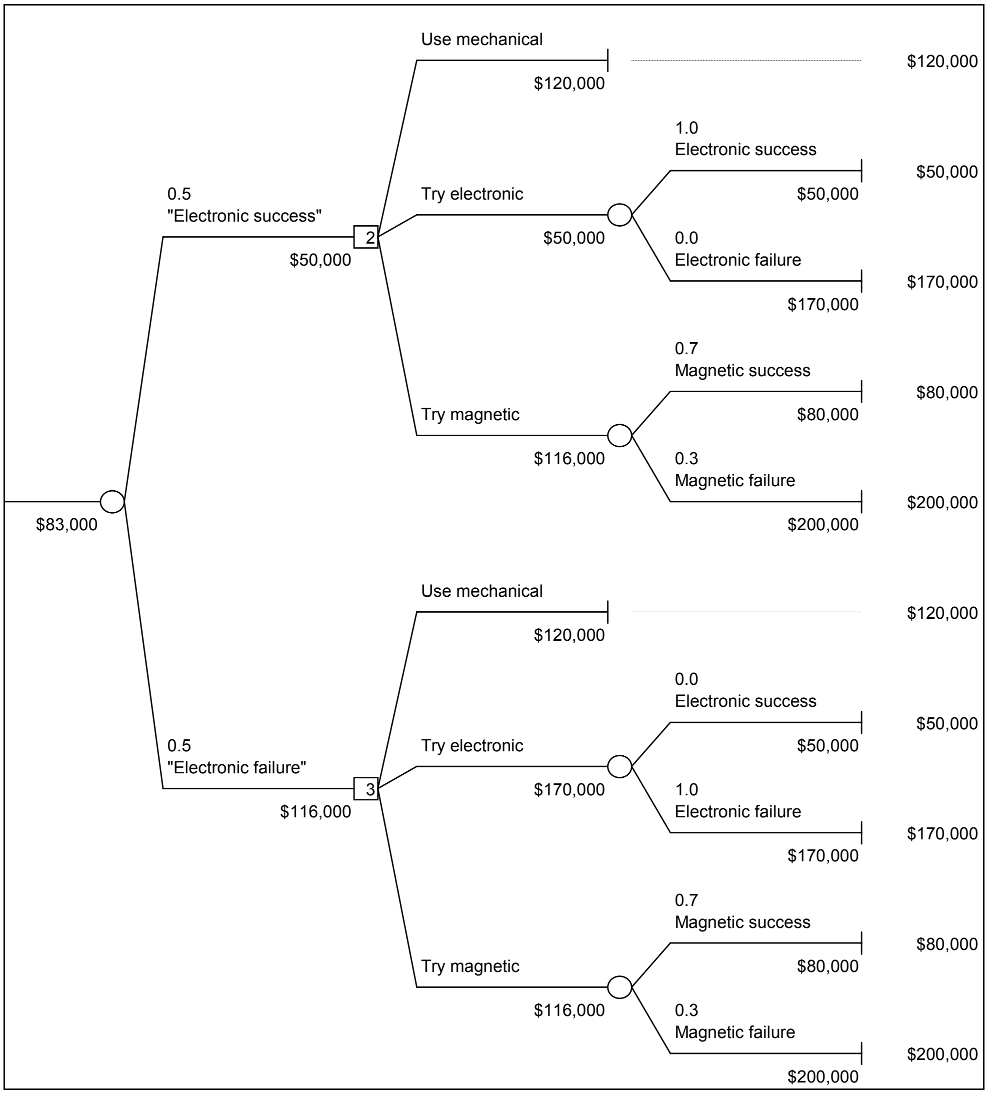


Figure 19.13 Expected Cost Under Uncertainty



ECUU = Expected Cost Under Uncertainty = $110,000

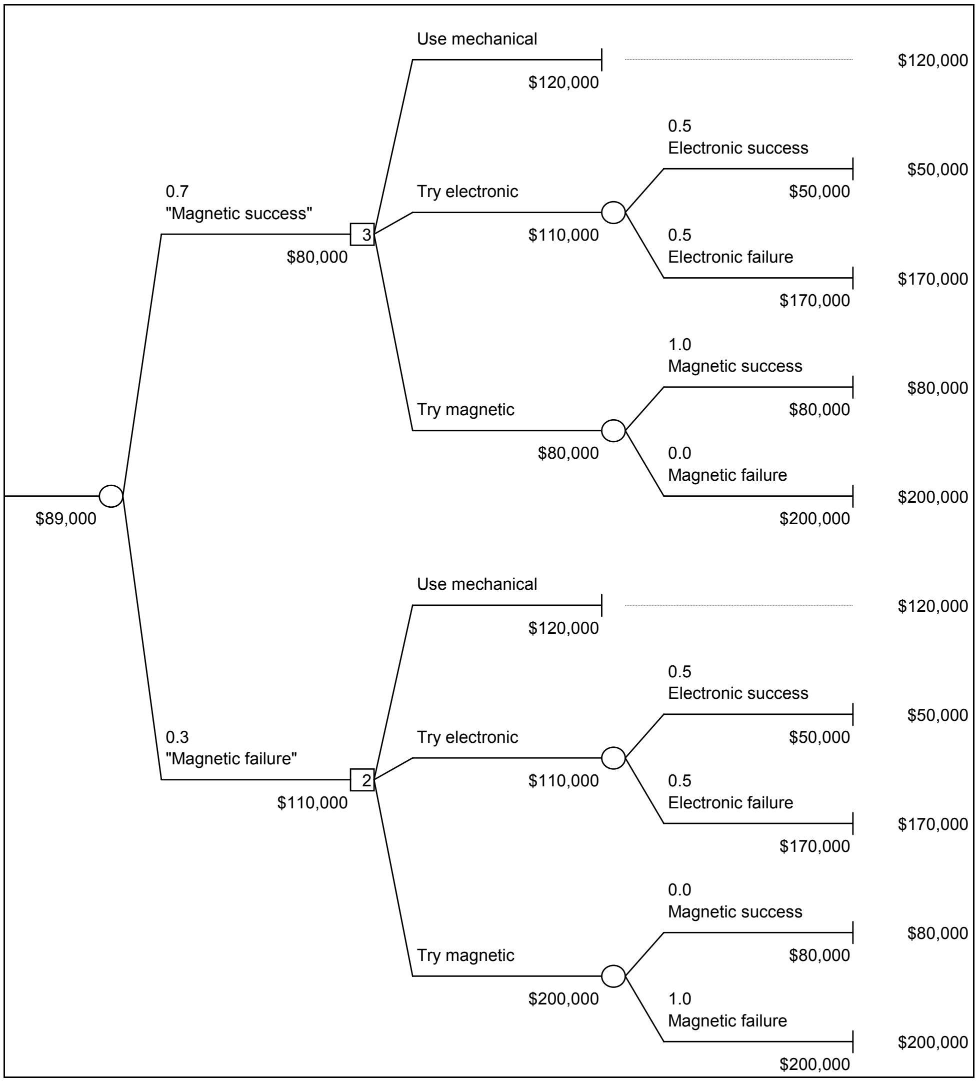
Figure 19.14 Expected Cost with Perfect Prediction for Electronic Uncertainty



ECPP(Electronic) = Expected Cost with Perfect Prediction for Electronic Uncertainty = $83,000

EVPI(Electronic) = ECUU – ECPP(Electronic) = $110,000 – $83,000 = $27,000

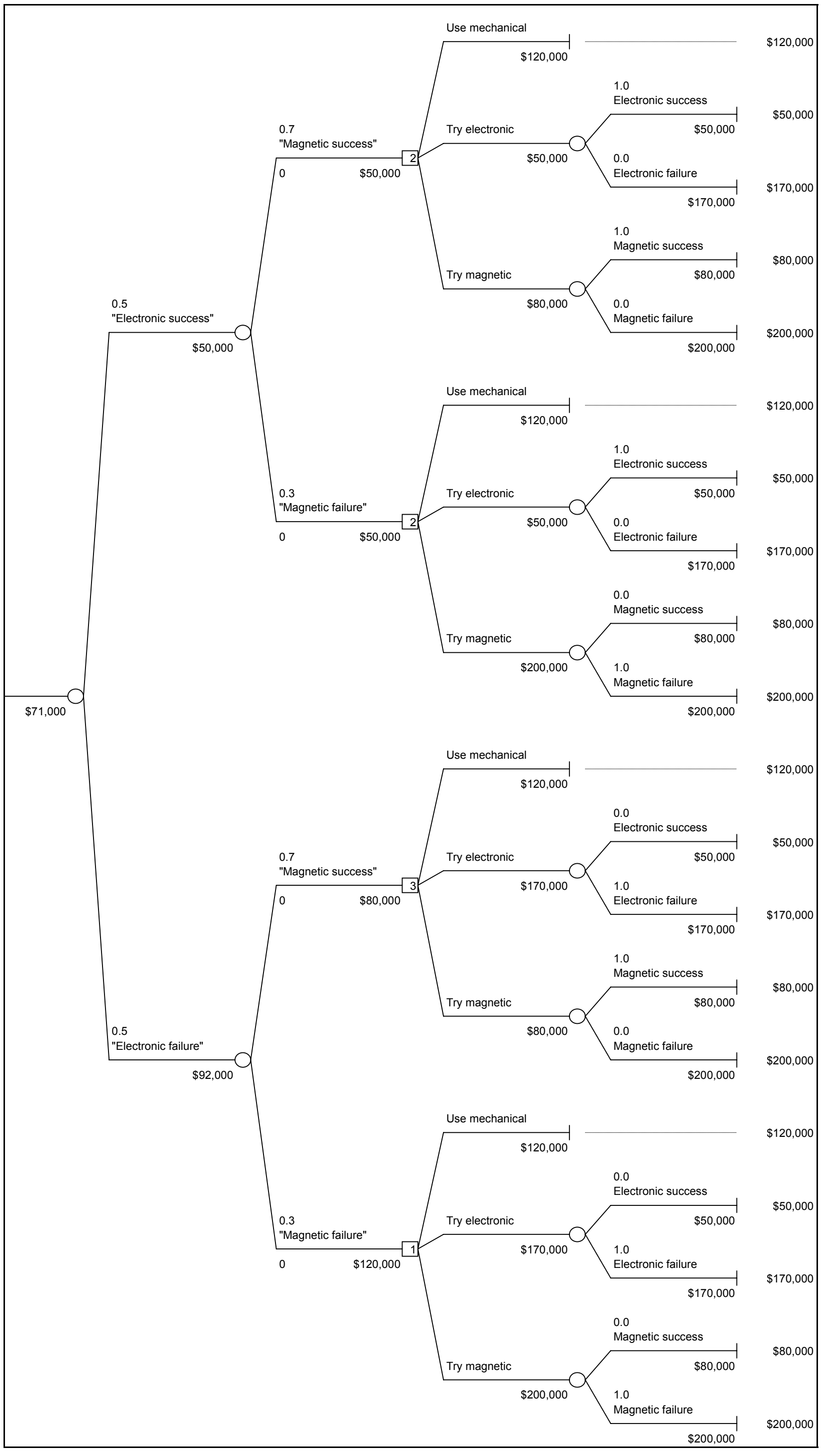
Figure 19.15 Expected Cost with Perfect Prediction for Magnetic Uncertainty



ECPP(Magnetic) = Expected Cost with Perfect Prediction for Magnetic Uncertainty = $89,000

EVPI(Magnetic) = ECUU – ECPP(Magnetic) = $110,000 – $89,000 = $21,000

Figure 19.16 Expected Cost with Perfect Prediction for Both Uncertainties



ECPP(Both) = Expected Cost with Perfect Prediction for Both Uncertainties = \$71,000

EVPI(Both) = ECUU – ECPP(Both) = \$110,000 – \$71,000 = \$39,000

EVPI(Electronic) + EVPI(Magnetic) = \$27,000 + \$21,000 = \$48,000

Here, EVPI(Both) ≠ EVPI(Electronic) + EVPI(Magnetic)

And, in general, as here, EVPIs are not additive.

In some special cases, EVPI(Two Events) = EVPI(First Event) + EVPI(Second Event)

19.4 SENSITIVITY ANALYSIS VS EVPI

Working Paper Title: Do Sensitivity Analyses Really Capture Problem Sensitivity? An Empirical Analysis Based on Information Value

Authors: James C. Felli, Naval Postgraduate School and Gordon B. Hazen, Northwestern University

Date: March 1998

The most common methods of sensitivity analysis (SA) in decision-analytic modeling are based either on proximity in parameter-space to decision thresholds or on the range of payoffs that accompany parameter variation. As an alternative, we propose the use of the expected value of perfect information (EVPI) as a sensitivity measure and argue from first principles that it is the proper measure of decision sensitivity. EVPI has significant advantages over conventional SA, especially in the multiparametric case, where graphical SA breaks down. In realistically sized problems, simple one- and two-way SAs may not fully capture parameter interactions, raising the disturbing possibility that many published decision analyses might be overconfident in their policy recommendations. To investigate the extent of this potential problem, we re-examined 25 decision analyses drawn from the published literature and calculated EVPI values for parameters on which sensitivity analyses had been performed, as well as the entire set of problem parameters. While we expected EVPI values to indicate greater problem sensitivity than conventional SA due to revealed parameter interaction, we in fact found the opposite: compared to EVPI, the one- and two-parameter SAs accompanying these problems dramatically overestimated problem sensitivity to input parameters. This phenomenon can be explained by invoking the flat maxima principle enunciated by von Winterfeldt and Edwards.

http://www.mccombs.utexas.edu/faculty/jim.dyer/DA_WP/WP980019.pdf